A Neo-Riemannian Approach to Jazz Analysis

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A Neo-Riemannian Approach to Jazz Analysis

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Neo-Riemannian theory originated as a response to the analytical issues surrounding Romantic music that was both chromatic and triadic while not “functionally coherent.”¹ This music retains some conventional aspects of diatonic tonality, though it stretches beyond the constraints as defined by earlier centuries. Richard Cohn outlines the difficulty in assigning a categorical label to this type of music. Firstly, the term “chromatic tonality” suggests pitch-centricity, which the music often lacks. Secondly, “triadic chromaticism” is also misleading in that it is too widely-encompassing of all chromatic harmony. Lastly, “triadic atonality” contradicts any tonal aspects of the music. After posing this conundrum, he offers the term “triadic post-tonality” (first suggested by William Rothstein) for much of the music composed in the

1. Nora Engebretsen and Per F. Broman, “Transformational Theory in the Undergraduate Curriculum: A Case for Teaching the Neo-Riemannian Approach,” *Journal of Music Theory Pedagogy* 21 (2007): 39.

latter portion of the nineteenth century.² Jazz music shares many of the same technical characteristics as Romantic music and, based on these stylistic similarities, it is highly plausible that neo-Riemannian techniques could prove to be highly valuable and effective when analyzing this newer genre of “triadic post-tonality.”

The neo-Riemannian approach builds upon portions of musicologist Hugo Riemann’s functional harmonic theories by applying them to non-functional chords. These chords do not interact with surrounding harmonies in a “key-defining manner” and do not rely on the resolutions that usually occur after “active” scale degrees like the leading tone.³ Neo-Riemannian theorists examine music using a transformational approach, meaning that a piece of music need no longer possess traditional diatonic root relationships in order to be analyzed.⁴ According to Jocelyn Neal, applying neo-Riemannian principles to newer genres of music such as popular music and jazz has “opened up compelling avenues for interdisciplinary research.”⁵ This “instinctive” merge began with a generation of music theorists who grew up with and celebrated “the supposed collapse between ‘high’ and ‘low’ culture” of classical music and popular music.⁶

2. Richard Cohn, “Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective,” *Journal of Music Theory* 42, no. 2 (Autumn 1998): 168, <http://jstor.org/stable/843871>.

3. Steven Strunk, “Notes on Harmony in Wayne Shorter’s Compositions, 1964-67,” *Journal of Music Theory* 49, no. 2 (2005): 303, <http://www.jstor.org/stable/27639402>.

4. Engebretsen and Broman, “Transformational Theory,” 40.

5. Jocelyn Neal, “Popular Music Analysis in American Music Theory,” *Zeitschrift Der Gesellschaft für Musiktheorie* 2, no. 2 (2005): 1, <http://www.gmth.de/zeitschrift/artikel/pdf/524.aspx>.

6. Ibid., 2.

Steven Strunk and Guy Capuzzo have been the main pioneers in exploring neo-Riemannian jazz analysis to date. Some of Strunk's first articles on the subject develop contextual operations for the analysis of seventh-chord progressions in post-bebop jazz while his article "Notes on Harmony in Wayne Shorter's Compositions, 1964-67" investigates the jazz harmonies of Shorter in particular.⁷ Taking a different approach, Capuzzo studies the instructional work "The Nature of Guitar" by Pat Martino and surveys the overlapping relations between the manual and neo-Riemannian principles.⁸ This paper differs from the works of Strunk and Capuzzo in that it aims to provide a broader view of the intersection of neo-Riemannian theory and standard popular songs from a handful of jazz subgenres. The following analyses examine lead sheets from the fifth edition of *The Real Book*, a detailed and widely respected collection of transcriptions of a primarily aural music. While the music may not have been composed by *using* neo-Riemannian techniques, it seems to hold the same core intent as the Romantic music for which the theory was created: to transcend the boundaries of traditional tonality through chromatic harmony and parsimonious voice leading. For this reason, I will explore the extent to which neo-Riemannian techniques can be applied to the analysis of standard mid-twentieth-century jazz repertoire.

The fundamental venture of neo-Riemannian theory is to investigate transformational relationships among the

7. Strunk, "Notes on Harmony in Wayne Shorter's Compositions," 301-332.

8. Guy Capuzzo, "Pat Martino and the 'Nature of the Guitar': An Intersection of Neo-Riemannian Theory and Jazz Theory," *Music Theory Online* 12, no. 1 (2006): 4, http://libres.uncg.edu/ir/f/G_Capuzzo_Pat_2006.pdf.

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twenty-four possible set class (SC) 037 triads – twelve major and twelve minor – in “algebraically elegant” and “musically suggestive” ways that can be visualized in various forms by the use of a graph called the *Tonnetz* (tone network).⁹ The *Tonnetz*, or Cartesian plane, allows common-tone retention and harmonic motion to be plotted spatially. This is particularly important to some of the theoretical perspectives that emerged in response to nineteenth-century music: namely, that triadic proximity could be examined through the number of shared common tones rather than through rigid tonality and root relation.¹⁰

The plane is designed so that each triangle represents a triad whose three points are representative of individual notes: a triangle with its point at the top is a major chord whereas one with its point facing down is a minor chord. It is constructed using three axes, each representing a different interval from point to point (or note to note). The horizontal axis is comprised of perfect fifth relationships (ex. $D \rightarrow A \rightarrow E...$); the axis travelling from bottom left to top right contains major third relationships (ex. $Bb \rightarrow D \rightarrow F\#...$); and the axis from bottom right to top left shows minor third relationships (ex. $Ab \rightarrow F \rightarrow D...$). One can then use the *Tonnetz* to map chords as they progress, as though there is a triangular object flipping along the various axes as the music transitions from harmony to harmony.¹¹

9. Julian Hook, “Exploring Musical Space,” *Science* 313, no. 49 (2006): 49, DOI: 10.1126/science.1129300.

10. Cohn, “Introduction to Neo-Riemannian Theory,” 174.

11. If one is to assume equally-tempered pitch classes, rather than just-intoned pitches, then Gollin (1998) clarifies that “the *Tonnetz* would be situated not in an infinite Cartesian plane but on the closed, unbounded surface of a [...] hyper-torus in 4-dimensional space.” While

Figure 1: *A standard Cartesian plane with L, P, and R transformations labelled (adapted from Cohn's Figure 2).¹²*

The *Tonnetz* unites the concept of trichords in transformational theory with those in nineteenth-century harmonic theory through its combination of triads and transformations.¹³ The three primary transformations in neo-Riemannian theory are mathematical operations – L, P, and R – that depict specific, pre-determined ways of transforming one chord into another.¹⁴ The *Leittonwechsel* relationship (L)

this is an important point in the study of neo-Riemannian theory, the 2-D Cartesian plane in Figure 1 is more appropriate and intuitive than the torus for deciphering small-scale transformations such as those in this article.

12. Cohn, “Introduction to Neo-Riemannian Theory,” 172.

13. Ibid.

14. Hook, “Exploring Musical Space,” 49. Lewin (among others) has discussed another transformation, “D”, as a valuable neo-Riemannian operation. I opt not to use it in my analyses for several reasons. First, it is not a true transformation, as a “V” chord can be the dominant of two different chords: “I” and “İ”. By definition, a transformation can only yield one outcome. Second, it is not a contextual inversion like L, P, and R. Third, the D transformation can occur from the dominant to the tonic

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takes the root of a major chord and moves it a semitone down to the leading tone, preserving the third and fifth; it preserves the root and third of a minor chord and shifts the fifth a semi-tone upward.¹⁵ The Parallel (P) retains the root and fifth, moving the third down a semitone from major to minor or up a semitone from minor to major. The Relative (R) moves the fifth of a major chord up a tone to become the root of its relative minor, or the root of the minor chord down a tone to become the fifth of its relative major. All of these transformations change the chord quality from major to minor or vice versa and can occur in either direction on the *Tonnetz*.

The Slide (S), a less commonly discussed transformation, maintains a stationary third while the root and fifth both slide up or down a semitone in similar motion. This changes the chord quality from major to minor or the reverse.¹⁶ When S is plotted on a *Tonnetz*, the triangle flips about a point on a horizontal axis. Using Figure 1, the C minor chord would retain the note E-flat as a common tone while the notes C and G flip about the axis to become the notes C-flat and G-flat, respectively. The Slide is a frequent transformation in jazz; it can be heard in Antonio Carlos

or in reverse and harmonically speaking, the functions of both are drastically different. Fourth, combining L and R in analyses usually presents a more accurate portrayal of what is occurring in the music – especially when travelling between two minor chords (which would rarely be tonally analyzed as dominant in function, anyway).

15. This can also be referred to as a Leading Tone relationship.

16. Lewin (1987) ascribed the label “SLIDE” to this operation, though Capuzzo (2004) refers to it as “P”. There are in fact three different types of Slide transformations - one for each axis of the *Tonnetz* - though my analyses only pertain to one, which I will call “S”.

Jobim's 'One Note Samba' (1961) and 'The Girl from Ipanema'(1963), shown in Figure 2. Since the *Tonnetz* is not suitable for seventh chords, as I will discuss further into the article, I have omitted the sevenths in my analysis.

Figure 2a: *Mm. 1-2 of 'One Note Samba.'*¹⁷

Figure 2b: *Slide transformation in mm. 1-2 of 'One Note Samba.'*

17. Antonio Carlos Jobim, "One Note Samba," *The Real Book*, 5th ed., (1988): 331.

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Figure 2c: *Mm. 5-6 of 'The Girl from Ipanema.'*¹⁸

Figure 2d: *Slide transformation in mm. 5-6 of 'The Girl from Ipanema.'*

While the above *Tonnetz* is a consistent method for modelling triadic transformations, Julian Hook presents a plane redesigned by Tymoczko that is more easily navigated in analysis. According to Hook, it successfully “relates the geometry of the spaces to the musical behaviour of the chords that inhabit them.”¹⁹ The entire plane is rotated and as a result, the perfect fifth relationship is no longer horizontal

18. Antonio Carlos Jobim, “The Girl from Ipanema,” *The Real Book*, 5th ed., (1988): 171.

19. Hook, “Exploring Musical Space,” 49.

but is instead diagonal; the note names are converted to integers, faded to grey, and written in a smaller font;²⁰ the chord names are embedded within each triangle; arrows now depict the transformational directions throughout the *Tonnetz*; and, finally, the transformations are colour-coded in the legend. A number of these modifications make jazz analysis more straightforward – specifically, the inclusion of chord names (written in a larger font than the individual integers which construct the chords) and the addition of colour allow for greater ease when travelling through the diagram. This ensures that the focus is on the triads and their transformations, rather than on individual pitches.

Figure 3 illustrates the visual proximity of chords and fluidity of the progression when using Tymoczko's *Tonnetz* design. I then use his *Tonnetz* to show a phrase from 'For Heaven's Sake' (1946), which exhibits a variety of single transformations as well as combinations. The chords are circled in orange on the *Tonnetz* and numbered in order of occurrence.²¹

Figure 3a: *Mm.* 13-18 of 'For Heaven's Sake.'²²

20. It is standard practice in certain types of music theory to represent the note C with the number 0, C# with 1, D with 2, and so forth.

21. Again, it was necessary to reduce the chords to triads due to the inherent limitations of the plane.

22. Elise Bretton, Sherman Edwards, and Donald Meyer, "For Heaven's Sake," *The Real Book*, 5th ed., (1988): 159.

Figure 3b: *An original analysis of 'For Heaven's Sake,' mm. 13-18, mapped on Hook's Tonnetz (his Figure 1).²³*

Along with the *Tonnetz*, Hook lists other contributions of neo-Riemannian theory that include a “fresh” perspective on the concepts of consonance, dissonance, symmetry, and efficient voice leading in composition.²⁴ Childs, however, remarks on a fundamental issue regarding the *Tonnetz* that has arisen in the last fifteen years or so: “the composers whose works seem best suited for neo-Riemannian analysis rarely limited their harmonic vocabulary to simple triads.”²⁵ In

23. Hook, “Exploring Musical Space,” 49.

24. Ibid., 50.

25. Adrian P. Childs, “Moving beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords,” *Journal of Music Theory* 42, no. 2 (Autumn 1998): 181, <http://jstor.org/stable/843872>. The fact that these composers do not limit themselves to triads may imply that they are not, in fact, well-suited for standard neo-Riemannian analysis and the *Tonnetz*. I interpret this statement to mean that the triadic foundations of these composers’ harmonic progressions move parsimoniously and in a way that can be plotted on a *Tonnetz*; in this way, they are ideal for this

approaching many of these composers' works with the neo-Riemannian tools currently at our disposal, theorists must often simplify chords that contain dissonances into triads. This often involves disregarding the sevenths of major-minor seventh chords and the root of half-diminished chords.²⁶ There is an inevitable loss with the simplification of seventh chords to triads; for example, one might choose to omit the seventh of a dominant seventh chord (scale degree 4 within the key) for the sake of showing the progression on a *Tonnetz*. By doing so, the strong downward semitone "pull" from scale degree 4 to scale degree 3 – an important aspect of a $V^7 \rightarrow I$ progression – is left unaccounted for.²⁷

Jazz music, in particular, is often rich with seventh chords and this increase in cardinality enables more relations to be formed between chords.²⁸ Adrian Childs highlights the

type of analysis. However, in order to map the chords, one must overlook the seventh. The result of omitting the seventh is incomplete analysis, in that it can never accurately represent the music that is actually there.

26. Childs, "Moving beyond Neo-Riemannian Triads," 182.

Major-minor seventh chords (SC 0258) contain a SC 037 triad within them. For example, CMm⁷ contains the notes C-E-G-Bb, with the first three notes C-E-G creating a major triad. This is why the seventh (Bb) is often omitted in neo-Riemannian analysis. Half-diminished chords are also of the set class 0258, and they too contain a SC 037 triad: F#^{o7} (F#-A-C-E) becomes A minor (A-C-E) when the root (F#) is omitted in analysis.

27. This is not to say that a simplification such as this can never be justified; if the important motion occurs in the voices that are retained after the reduction, it can be a logical analytical choice. For that reason, I occasionally choose to reduce chords to their triadic foundation in my analyses.

28. Edward Gollin, "Some Aspects of Three-Dimensional 'Tonnetze'," *Journal of Music Theory* 42, no. 2 (Autumn 1998): 196, <http://www.jstor.org/stable/843873>. Cardinality refers to the number of elements in a set. In this case, it refers to the number of individual pitches

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need for “a transformational system for dominant and half-diminished seventh chords which would allow all four pitches to participate in parsimonious voice leading.”²⁹ In 1998, Childs and Edward Gollin both design three-dimensional (3-D) models to accommodate certain tetrachords.³⁰ While Childs’ provides a more elaborate design, Gollin’s revamped 3-D *Tonnetz* (pictured in Figure 4) is more straightforward to navigate; therefore, it may be more useful for mapping short harmonic passages. The figure shows a central tetrachord prism in the center, with the notes C, E, G, B-flat. From each of the prism’s six edges stems one other tetrachord that shares two of the same notes from the common edge. This figure is particularly useful in demonstrating the inversive relationship between two tetrachords of the same set class but of a different mode with two notes in common.³¹ Gollin refers to this operation as the “S-transformation” which retains two pitches as common tones while the remaining two pitches move by half step in similar motion, shown in Figure 4.³² Not pictured below is another operation, “C-

in a chord: a C major chord has a cardinality of three (reflective of the notes C, E, and G), while a C Mm⁷ chord has a cardinality of four (C, E, G, and Bb).

29. Childs, “Moving beyond Neo-Riemannian Triads,” 185.

20. Gollin, “Some Aspects of Three-Dimensional ‘Tonnetz’,” 195.

31. There is an entirely different model for representing same-SC tetrachords with *one* note in common, also on page 201 of his article. While this, too, can easily lend itself to jazz analysis, I chose to omit it based on the relevance of Figure 4 to my argument, and the redundancies which would transpire.

32. Childs, “Moving beyond Neo-Riemannian Triads,” 185.

transformation,” which pertains to moving two of the four pitches a semitone in contrary motion.³³

Figure 4: *Gollin’s design of six ‘edge flips’ about a nexus tetrachord, (C,E,G,Bb), within an [0258] Tonnetz (his Figure 4b).*³⁴

Childs explains that a seventh-chord model for mapping harmonies is “more powerful” in neo-Riemannian analysis due to its accurate tracking of all four voices, rather than the current limited system of triadic transformations.³⁵ This type of *Tonnetz* is certainly innovative, though it is also quite limiting for the purpose of jazz analysis since the repertoire often features a variety of tetrachords types. The major seventh chord (0158), diminished seventh chord (0369)

33. Childs, “Moving beyond Neo-Riemannian Triads,” 185.

34. Gollin, “Some Aspects of Three-Dimensional ‘Tonnetze,’” 201.

35. Childs, “Moving beyond Neo-Riemannian Triads,” 191.

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and any 0258 tetrachord transposed from the CMm⁷ listed in Figure 4 requires an entirely new diagram. This figure simply cannot accommodate a wide enough range of chords as it is. As a result, I was required to transpose Gollin's original framework by T₅ so that it reflected the following excerpt of Charles Mingus' 'Fables of Faubus' in Figure 5.

Figure 5a: *Mm. 37-38 of 'Fables of Faubus'.³⁶*

Figure 5b: *Gollin's original S-transforms Tonnetz transposed by T₅ to express the Iⁱⁱⁱ_{iv} relationship from mm. 37-38 of 'Fables of Faubus'.³⁷*

There is a Iⁱⁱⁱ_{iv} relationship between FMm⁷ and C^{o7} chords Figure 5. The two chords invert (I) so that two points

36. Charles Mingus, "Fables of Faubus," *The Real Book*, 5th ed., (1988): 145-6.

37. Gollin, "Some Aspects of Three-Dimensional 'Tonnetze'," 201.

– the third ($^{\text{iii}}$) note of the FMm⁷, C, and the fourth ($_{\text{iv}}$) note, E-flat – map onto one another. These two inversion points are stationary pitches, shown in orange. The other two pitches (G-flat and B-flat) begin as the colour red but transform through the $I_{\text{iv}}^{\text{iii}}$ relationship to become the pitches shown in yellow (A and F).

The three-dimensional model is limiting in that it does not accommodate near-transformations. For example, the two moving voices must move by semitone; if one travels by semitone and the other by whole tone, the entire system is rendered ineffective. Voice leading by whole tone often occurs in jazz and using only Gollin's system to analyze seventh chords would severely limit its potential in the analysis of the genre as a whole. Joseph Straus presents a more flexible analytical system to accommodate transformations – most commonly transposition and inversion – that do not quite fit the standard, rigid mould.³⁸ These “fuzzy transformations,” originally presented by Ian Quinn (1997), entail examining each voice's individual transformation between simultaneities and then selecting the best overall transformation to use. This decision can be made by selecting what is essentially the median, mean, or mode (in the mathematical sense) of all the separate transformations. According to Straus: “the connections created by such fuzzy transpositions [or inversions] may serve to link harmonies that would be judged as incomparable by traditional, crisp

38. Joseph N. Straus, “Uniformity, Balance, and Smoothness in Atonal Voice Leading,” *Music Theory Spectrum* 25, no. 2 (Autumn 2003): 318, <http://www.jstor.org/stable/3595434>.

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atonal set theory.”³⁹ In this way, the “fuzzy transformation” is similar to a line of best fit for a graph, wherein the outliers are accounted for through the concept of offset. The offset is calculated by adding the total number of semitones the outliers would need to shift up or down to be an exact (or “crisp”) transformation.⁴⁰ In other words, one could measure the distance from each graphical outlier to the line of best fit and decipher the offset from the sum of these measurements.

‘One Note Samba’ contains various fuzzy inversions between a number of its seventh chords. For example, Figure 6 outlines a near- I_{ii}^i relationship in mm. 10-11 of the piece.⁴¹ In an ideal and exact transformation, this would be expressed by Gollin’s 3-D *Tonnetz* as an S-transformation; in actuality, only one chord is of the 0258 set class. This then creates an I_{ii}^i S-transformation with an offset of 1. The offset is illustrated at the bottom of Figure 6 while the shaded outlier (E-flat) and expected transformational output (E-natural) appear on the staff

39. Joseph N. Straus, “Uniformity, Balance, and Smoothness in Atonal Voice Leading,” *Music Theory Spectrum* 25, no. 2 (Autumn 2003): 318, <http://www.jstor.org/stable/3595434>

40. *Ibid.*, 315.

41. Again, this means that the inversion maps the lowest note ($\hat{i}$) of the original BbMm⁷ chord, Bb, onto the second-lowest note ($\hat{ii}$), G, and vice versa.

Figure 6: *Fuzzy S-transforms: I_{ii}^i with an offset of 1 between $Bb\ Mm^7$ and $Eb\ Maj^7$ in ‘One Note Samba,’ mm. 10-11.⁴²*

While highly practical in showing three possible relationships between major and minor chords that share common tones, the *Tonnetz* is but one method of spatially representing music.⁴³ Its limitations include its inability to properly accommodate chords of cardinality greater than three (or four, as in the cases of Gollin and Childs), diminished or augmented chords, or other important patterns in triadic movement like near-transformations. It also fails to properly recognize transformations that occur between objects of differing cardinalities; jazz is replete with passages that include both seventh chords and triads. Although it is beyond the scope of my study, recent work on cross-type transformations might provide a useful resource for modelling such progressions.⁴⁴

42. Jobim, “One Note Samba,” 331.

43. Hook, “Exploring Musical Space,” 50.

44. Hook (2007) in particular has developed methods of mapping triads onto seventh chords.

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Louis Bigo, Antoine Spicher, and Olivier Michel present a hexagonal lattice that uses many of the same principles as the standard neo-Riemannian triangular lattice but rephrases some of its algorithmic problems in spatial terms.⁴⁵ For example, the traditional *Tonnetz* is rephrased by placing each note in a hexagonal frame and aligning it diagonally and vertically surrounding notes to create intervallic patterns. The theorists' elaboration of the current triad-based system tolerates more complex arrangements in music. In Figure 7 it is still possible to see the various motions found in the three directions of a triangular *Tonnetz*: perfect fifth (horizontal), semitone (vertical) and thirds (diagonals). Since each note now has contact with six other notes instead of three, however, it drastically increases the number of possible relationships. Both major and minor triads still form triangular shapes, but diminished and augmented chords, as well as chords of any cardinality, can now be spatially obtained and compared with ease.

45. Louis Bigo, Antoine Spicher, and Olivier Michel, "Spatial Programming for Music Representation and Analysis," *Spatial Computing Workshop*, Budapest: Laboratoire d'Algorithmique, Complexité et Logique (2010): 5.

Figure 7: *A hexagonal lattice, rotated and adapted from Bigo, Spicher, and Michel's Figure 4.*⁴⁶

According to Bigo: “a musical piece can be seen as a spatial behaviour taking place on a spatial representation of notes... the whole progression corresponds to an ordered sequence of collections.”⁴⁷ In ‘One Note Samba,’ there is an ordered sequence whose pattern – despite being especially

46. Bigo, Spicher and Michel, “Spatial Programming,” 3.

47. Ibid., 4.

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smooth – cannot be adequately exposed using the traditional *Tonnetz*. The pattern becomes highly relevant when using Bigo's hexagonal lattice. Figure 8 depicts the progression where, by reducing each chord to its most basic and audible triadic framework, the consistent T_{11} pattern is shown by a gradual downward shift in the lattice. I omitted the added sixth on the first chord and the sevenths on the second and third chords, since they are all B-flat – the 'one note' of the 'One Note Samba.'⁴⁸ This sustained note of course creates parsimony in the overall phrase, but I am more interested in examining the motion of the supporting harmonies.

Figure 8a: *Mm. 29-32 of 'One Note Samba.'*⁴⁹

48. In this example, the B♭ acts as a pedal tone. Though it demonstrates parsimony throughout the passage, it is not part of the semitone motion that I wish to expose with Figure 8a. I also omitted the sixth on the B♭ major chord to better reflect the overall pattern occurring in the harmonies.

49. Jobim, "One Note Samba," 331.

Figure 8b: ‘One Note Samba,’ *mm.* 29-32, as presented on a hexagonal lattice.⁵⁰

Figure 9 explores the hexagonal lattice as being capable of showing common-tone retention between chords with cardinalities greater than three. While the chords in Gershwin’s ‘A Foggy Day’ are parsimonious, they demonstrate added tones rather than patterned movement as in Figure 8. The sevenths on all three chords, along with the 9th and flat 5th, could not be adequately shown using a standard *Tonnetz* and would have to be discarded if using the plane. By so doing, the important semitone motion from E to E-flat between the first two chords (and the subsequent semitone motion from G to F-sharp between the second and third chords) would also be sacrificed.

50. Bigo, Spicher and Michel, “Spatial Programming,” 4.

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Figure 9a: *Mm. 1-2 of 'A Foggy Day.'*⁵¹

Figure 9b: *'A Foggy Day,' mm.1-2, as presented on a hexagonal lattice.*

David Rappaport has represented yet another relationship between music and geometry through the use of combinatorics. To show the musical progressions, Rappaport

51. George Gershwin, "A Foggy Day," *The Real Book*, 5th ed., (1988): 6.

uses an atonal clock: a circle with twelve equidistant points which together represent the twelve chromatic pitches, beginning with 0 (C) at the topmost point.⁵² From there, Rappaport connects certain subsets of pitches within the circle to form geometric shapes. It is highly practical to visualize scales and chords in this way since the transposition (rotation) and inversion (flip) of these shapes become easily detectable. Figure 10 presents the clock wherein lies the diminished seventh chord, whose points both are symmetrical and as evenly spaced in the circle as possible; thus, the diagram is considered ‘maximally even’.⁵³

Figure 10: *Rappaport’s combinatorics representing the diminished seventh chord (from his Figure 5).*⁵⁴

The augmented triad is also maximally even and has been featured in a number of musical cycles. Dan Adler describes a musical cycle as “an ordered set of notes obtained

52. David Rappaport, “Geometry and Harmony,” *Proceedings of BRIDGES: Mathematical Connections in Art, Music, and Science*, Banff, Alberta (2005): 1, DOI: 10.1.1.72.2002. It is important to highlight that the circular nature of this system relies upon equal temperament in the music; without it, the tuning between notes in an octave may vary such that a number on the atonal clock may no longer accurately represent the note.

53. Rappaport, “Geometry and Harmony,” 2.

54. *Ibid.*, 5.

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by successively applying the same interval.”⁵⁵ In other words, when pitches or simultaneities undergo the same transformation repeatedly, they will eventually return to the starting chord and complete the cycle.⁵⁶ The augmented cycle, based upon the M3/m6 formula, transposes the root of each chord clockwise by a major 3rd (or T₄).⁵⁷ This combination creates four possible cycles of three notes each, as shown in Figures 11a and 11b. This pattern, highly prevalent in the works of John Coltrane, has since been labelled as the “Coltrane changes” and is often associated with the tune ‘Giant Steps’.⁵⁸

Like Rappaport, Adler also expresses the four augmented triads using geometry. While he shows all four possibilities of augmented chords, having each triangle’s points face the same way contradicts Rappaport’s precisely positioned Combinatorics circle. I combine and slightly modify these illustrations (using Adler’s labels on the circle with Rappaport’s clear circular outline) with that of Capuzzo, who in his article “Pat Martino and the ‘Nature of Guitar’” does in fact show the rotation of the augmented triad to span all twelve notes. Remaining consistent with the standard atonal clock, the top-most point on my figure is always 0, or the note C, to ensure visual consistency. I also assign a different colour to each of the four augmented cycles to more easily distinguish the intervallic relationships and

55. Dan Adler, “The ‘Giant Steps’ Progression and Cycle Diagrams,” *Jazz Improv Magazine* (Eric Nemeyer) 3, no. 3 (1999): 1, <http://danadler.com/misc/Cycles.pdf>.

56. These are often referred to as “sequences” in tonal repertoire.

57. Adler, “Cycle Diagrams,” 3.

58. Ibid., 3 and 7.

transpositions when I combine the cycles at the center of the figure.

Figure 11a: *Adler's four instances of the augmented cycle's M3/m6 motion (his Figure 6).*⁵⁹

59. Adler, "Cycle Diagrams," 7.

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Figure 11b: *Capuzzo's presentation of the augmented triangle and its transpositions (his Example 1).*⁶⁰

Figure 11c: *An adaptation of the ideas of Rappaport, Adler, and Capuzzo to create an original representation of the augmented cycle through combinatorics.*

60. Capuzzo, “Neo-Riemannian Theory and Jazz,” 4.

The yellow triangle in Figure 11c represents the exact pitches – B, G, and E-flat – upon which Coltrane elaborates when he tonicizes each chord by its individual dominant in ‘Giant Steps,’ shown with blue arrows in Figure 12.⁶¹ Travelling counter-clockwise with these two factors in mind yields the following progression in the piece’s introduction:

Figure 12: *The embellished Augmented Cycle from mm. 1-7 of Coltrane’s ‘Giant Steps.’*⁶²

B - D7 | G - Bb7 | Eb ... F#7 | B ||

The figure displays musical notation for the introduction of Coltrane's 'Giant Steps'. It consists of two staves of music. The first staff shows the chords B, D7, G, Bb7, Eb, and F#7. The second staff shows the chords G, Bb7, Eb, F#7, and B. Some chords are highlighted in red. To the right of the notation is a chromatic scale diagram, a circle with 12 positions labeled 0 through 11. The positions are: 0 (e), 1 (f), 2 (g), 3 (a), 4 (bb), 5 (c), 6 (db), 7 (e), 8 (f), 9 (g), 10 (a), 11 (bb). A yellow triangle connects positions 0, 2, and 7. Blue arrows indicate a counter-clockwise path from 0 to 2 to 7 to 0. A dashed blue line connects positions 0 and 7.

I chose to omit the chords shown in red to better highlight the complete augmented thirds cycle (shown in bold), since the last four of the omitted chords show exact repetition of previous material and the Am⁷ chord is simply a slight variation. The augmented cycles by nature generate complex harmonies efficiently while using parsimonious voice leading and symmetry that, as shown above, can be easily linked to combinatorics.⁶³

61. Capuzzo, “Neo-Riemannian Theory and Jazz,” 8.

62. John Coltrane, “Giant Steps,” *The Real Book*, 5th ed., (1988):

170.

63. Capuzzo, “Neo-Riemannian Theory and Jazz,” 9 and 16.

Nota Bene

Though neo-Riemannian theory owes its origin to late-nineteenth-century music, recent developments in the theory have made it more inclusive of and applicable to popular genres. Jazz music is particularly well suited to its specific techniques as the genre exhibits similar behaviours to those of the Romantic period, including a daring shift from tonality and a focus on common-tone preservation.

With its reliable system for analysis and ever-increasing efficiency, neo-Riemannian theory has become a vessel through which jazz can be examined. The hypothetical torus shape of the *Tonnet* _{$\mathbb{Z}$} , though not pictured in this essay, also allows for ordered visual representation of the various R, L, P, and S transformations; it represents harmonic shifts through space and time. Gollin further expands the *Tonnet* _{$\mathbb{Z}$} to accommodate for cardinalities greater than three. His representation of transformations among tetrachords is particularly useful when examining jazz music; indeed, dominant sevenths and half-diminished sevenths are some of the most common chords to occur. However, there are some aspects of the *Tonnet* _{$\mathbb{Z}$} that do not accurately portray the patterns in this music. The hexagonal lattice forges more relationships between notes by placing notes in the shape of a hexagon. By so doing, it creates a geometric representation that can better portray the vast possibilities of harmonic movement.

Other relationships have also been fused between geometry and music, including Rappaport's Combinatorics and the emerging field of cross-type transformations (not discussed in this article) to which Hook has made significant contributions by accommodating chords of different

cardinalities through various means.⁶⁴ Since harmonic progressions in jazz usually contain instances of different cardinalities among its constituents, further exploratory research in the field of cross-type transformations could provide new insight into ways of modelling songs of this genre. Other topics for future research include improvisation styles in each of the specific jazz subgenres (like bebop, bossa nova and cool jazz), differences in jazz voice leading according to instrument, and a more detailed exploration of twenty-first century jazz and jazz fusion artists.

With the innovative explorations of chordal space presented in this article, neo-Riemannian theorists have inadvertently shaped a system that is extremely compatible with the study of jazz.⁶⁵ It is my hope that this essay has fashioned an overlap of mutual gain between two academic circles not commonly associated with one another and suggested the value of transformational and neo-Riemannian theory in jazz analysis.

64. Julian Hook, "Cross-Type Transformations and the Path Consistency Condition," *Music Theory Spectrum* 29, no. 1 (Spring 2007): 1-40, <http://jstor.org/stable/10.1525/mts.2007.29.1.1>.

65. Hook, "Exploring Musical Space," 49.

Nota Bene

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